Seesaw mechanism and the neutrino mass matrix

E. Kh. Akhmedov[a]

[a] Centro de Física das Interacções Fundamentais (CFIF)
Departamento de Física, Instituto Superior Técnico
Av. Rovisco Pais, P-1049-001, Lisboa, Portugal

The seesaw mechanism of neutrino mass generation is analysed under the following assumptions: (1) minimal seesaw with no Higgs triplets, (2) hierarchical Dirac masses of neutrinos, (3) large lepton mixing primarily or solely due to the mixing in the right-handed neutrino sector, and (4) unrelated Dirac and Majorana sectors of neutrino masses. It is shown that large mixing governing the dominant channel of the atmospheric neutrino oscillations can be naturally obtained and that this constrained seesaw mechanism favours the normal mass hierarchy for the light neutrinos leading to a small U_{e3} entry of the lepton mixing matrix and a mass scale of the lightest right handed neutrino $M \simeq 10^{10} - 10^{11}$ GeV. Any of the three main neutrino oscillation solutions to the solar neutrino problem can be accommodated. The inverted mass hierarchy and quasi-degeneracy of neutrinos are disfavoured in our scheme. This talk is based on the work [1].

1. INTRODUCTION

In this talk, I discuss how phenomenologically viable neutrino masses and mixings can be generated within the framework of the the seesaw mechanism of neutrino mass generation, constrained by the following set of assumptions:

(i) Three generation $SU(2)_L\times U(1)$ model with the addition of three right-handed neutrino fields, which are singlets under $SU(2)_L\times U(1)$. No Higgs triplets are introduced and thus the effective mass matrix for the left-handed Majorana neutrinos is entirely generated by the seesaw mechanism, being given by

$$m_L = -m_D M_R^{-1} m_D^T\,, \qquad (1)$$

where m_D and M_R denote the neutrino Dirac mass matrix and the Majorana mass matrix of right-handed neutrinos.

(ii) The neutrino Dirac mass matrix m_D has a hierarchical eigenvalue structure, analogous to the one for the up-type quarks.

(iii) Charged lepton and neutrino Dirac mass matrices, m_l and m_D, are "aligned" in the sense that in the absence of the right-handed mass M_R, the leptonic mixing would be small, as it is in the quark sector. In other words, we assume that the *left-handed* rotations that diagonalize m_l and m_D are the same or nearly the same. We therefore consider that the large lepton mixing results from the fact that neutrinos acquire their mass through the seesaw mechanism.

(iv) The Dirac and Majorana neutrino mass matrices are unrelated.

2. GENERAL FRAMEWORK

Under our assumptions, in the basis where the mass matrix of charged leptons has been diagonalized, the effective mass matrix of light neutrinos m_L can be written as

$$\begin{pmatrix} m_u^2 M_{11}^{-1} & m_u m_c M_{12}^{-1} & m_u m_t M_{13}^{-1} \\ m_u m_c M_{12}^{-1} & m_c^2 M_{22}^{-1} & m_c m_t M_{23}^{-1} \\ m_u m_t M_{13}^{-1} & m_c m_t M_{23}^{-1} & m_t^2 M_{33}^{-1} \end{pmatrix} \qquad (2)$$

Here $M_{ij}^{-1} \equiv (M_R')_{ij}^{-1}$, where M_R' is the mass matrix of the right handed neutrinos in the basis where the neutrino Dirac mass matrix m_D is diagonal, and m_u, m_c and m_t are the eigenvalues of m_D. For our numerical estimates we take them to be equal to the masses of the corresponding up-type quarks, but for our general arguments their precise values are unimportant. The mass matrix (2) has to be compared with the phenomenologically allowed neutrino mass matrices.

Consider first the direct neutrino mass hierarchy $m_1, m_2 \ll m_3$. Assuming that $\theta_{23} \simeq 45°$ which is the best fit value of the Super-Kamiokande data [2], and taking into account that the CHOOZ experiment indicates that $\theta_{13} \ll 1$ [3], one can show that m_L must have the approximate form [4]

$$m_L = m_0 \left(\begin{array}{ccc} \kappa & \varepsilon & \varepsilon' \\ \varepsilon & 1+\delta-\delta' & 1-\delta \\ \varepsilon' & 1-\delta & 1+\delta+\delta' \end{array} \right), \quad (3)$$

where κ, ε, ε', δ and δ' are small dimensionless parameters. Comparing (2) and (3), one concludes that the following relations should hold, in leading order:

$$m_c^2\, M_{22}^{-1} = m_t^2\, M_{33}^{-1} = m_c m_t\, M_{23}^{-1}\,. \quad (4)$$

These relations seem to indicate that in order to obtain the form of Eq. (3), strong correlations are required between the eigenvalues of m_D and the entries of $(M_R')^{-1}$, in apparent contradiction with our assumption (iv). However, in fact there is no conflict. Obviously, the form of $(M_R')^{-1}$ depends on the ν_R basis one chooses. We have defined M_R' in the basis where m_D is diagonal, i.e. have included into its definition the right-handed rotation arising from the diagonalization of m_D. Therefore $(M_R')^{-1}$ contains information about the Dirac mass sector, and Eq. (4) is not necessarily in conflict with our assumption (iv). This assumption has to be formulated in terms of weak-basis invariants. What should be required is that the ratios of the *eigenvalues* of $(M_R')^{-1}$ should not be related to the ratios of the *eigenvalues* of m_D.

In order to see how the phenomenologically favoured form of m_L can be achieved without contrived fine tuning between the parameters of the Dirac and Majorana sectors, let us first consider the two-dimensional sector of m_L in the 2-3 subspace, which is responsible for a large θ_{23}. We shall write the diagonalized Dirac mass matrix m_D^{diag} as

$$m_D^{diag} = m_t\, diag(p^2 q\,,\ p\,,\ 1), \quad p = m_c/m_t \sim 10^{-2},$$
$$q = m_u m_t/m_c^2 \sim 0.4\,. \quad (5)$$

The 2-3 sector of $(M_R')^{-1}$, in order to lead to the 2-3 structure of Eq. (3) (with all elements approximately equal to unity up to a common factor), should have the following form:

$$M_R^{-1} \propto \left(\begin{array}{cc} 1 & p \\ p & p^2 \end{array} \right). \quad (6)$$

The eigenvalues of the matrix in Eq. (6) are 0 and $1+p^2$, and thus by choosing the pre-factor to be $const/(1{+}p^2)$ one arrives at the matrix M_R^{-1} of the desired form with p- and q-independent eigenvalues. It turns out to be possible to find a 3×3 matrix $(M_R')^{-1}$ with p- and q-independent eigenvalues, whose 2-3 sector generalizes (6) so as to obtain the realistic form of m_L in (3) with $\delta, \delta' \neq 0$ [1]:

$$(M_R')^{-1} = S_R^T\, (M_R^0)^{-1}\, S_R \quad (7)$$

with

$$S_R = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & c_\phi & s_\phi \\ 0 & -s_\phi & c_\phi \end{array} \right), \quad (8)$$

where $c_\phi = \cos\phi$, $s_\phi = \sin\phi$, $\phi = \arctan p$, and

$$(M_R^0)^{-1} = \frac{1}{2M} \left(\begin{array}{ccc} \gamma & \beta & \alpha \\ \beta & 1 & 0 \\ \alpha & 0 & r \end{array} \right). \quad (9)$$

The dimensionless parameters α, β, γ and r in (9) do not depend on p and q. Eqs. (7) - (9) yields the following mass matrix for the light neutrinos:

$$m_L \simeq \frac{m_t^2}{2M} \frac{p^2}{1+p^2} \times$$
$$\left(\begin{array}{ccc} q'^2 p^2\, \gamma & q' p\,(\beta-\alpha p) & q'\,(\alpha+\beta p) \\ q' p\,(\beta-\alpha p) & 1-r/4p^2 & 1-r/4p^2 \\ q'\,(\alpha+\beta p) & 1-r/4p^2 & 1+3r/4p^2 \end{array} \right) (10)$$

Here $q' \equiv q\sqrt{1+p^2} \simeq q$. Thus, we have obtained m_L of the desired form, while abiding by our assumptions. The particular case of the neutrino mass matrix of the form (10) with $\beta=\gamma=r=0$ (which allows only the vacuum oscillations solution of the solar neutrino problem) was obtained in [5].

Comparison of Eqs. (3) and (10) allows one to express the parameters of the phenomenological mass matrix of light neutrinos κ, ε, ε' δ and δ'

in terms of the parameters of the mass matrix of the right handed neutrinos α, β, γ and r and the parameters of the Dirac mass matrix m_t, p and q. The largest eigenvalue of the matrix m_L in (10) is

$$m_3 \simeq \frac{m_t^2}{M}\frac{p^2}{1+p^2} \simeq \frac{m_c^2}{M}\,. \qquad (11)$$

It scales as m_c^2 rather than as usually expected m_t^2. It has to be identified with $\Delta m^2_{atm} \simeq (2-6)\times 10^{-3}$ eV^2, which gives $M \simeq 10^{10}-10^{11}$ GeV, i.e. an intermediate mass scale rather than the GUT scale.

3. INVERTED MASS HIERARCHY AND QUASIDEGENERACY

Direct inspection of the neutrino mass textures that lead to the inverted mass hierarchy $m_3 \ll m_1 \simeq m_2$ and the quasi-degenerate case with $m_1 \simeq m_2 \simeq m_3$ show that all of them except one do not satisfy our assumptions (i)-(iv) [1]. In particular, for these textures it is impossible to have both traces $(\Lambda_1+\Lambda_2+\Lambda_3)$ and second invariants $(\Lambda_1\Lambda_2+\Lambda_1\Lambda_3+\Lambda_2\Lambda_3)$ of the corresponding matrices $(M_R')^{-1}$ to be p- and q-independent (here Λ_i $(i=1,2,3)$ are the eigenvalues of $(M_R')^{-1}$). The remaining texture corresponds to the mass matrix of the heavy singlet neutrinos M_R' with the following eigenvalues: two singlet neutrinos are almost degenerate with $M_1 \simeq M_2 \sim 10^8$ GeV; the third mass eigenvalue turns out to be well above the Planck scale: $M_3 \sim 10^{22}$ GeV, clearly not a physical value. Thus, this case of the inverted mass hierarchy is ruled out as well.

4. DISCUSSION

We have shown that the seesaw mechanism, supplemented by the set of assumptions listed in the Introduction, leads to phenomenologically viable mass matrices of light active neutrinos. The mixing angle θ_{23} responsible for the dominant channel of the atmospheric neutrino oscillations can be naturally large without any fine tuning.

We have found that all three main neutrino oscillations solutions to the solar neutrino problem – small mixing angle MSW, large mixing angle MSW and vacuum oscillations – are possible within the constrained seesaw. The numerical examples of the requisite values of the parameters of the Majorana mass matrix M_R' of singlet neutrinos are given in [1].

Although the constrained seesaw mechanism allows to obtain a large mixing angle θ_{23} in a very natural way, it does not explain why θ_{23} is large: the largeness of this mixing angle is merely related to the choice of the inverse mass matrix of heavy singlet neutrinos, Eq. (7) - (9). However, once this choice has been made, the smallness of the mixing angle θ_{13} which determines the element U_{e3} of the lepton mixing matrix can be readily understood. For the case of the normal mass hierarchy $m_1, m_2 \ll m_3$ the value of θ_{13} can be expressed in terms of the entries of the effective mass matrix m_L in Eq. (3) as $\sin\theta_{13} \simeq (\varepsilon+\varepsilon')/2\sqrt{2}$ [4]. One then finds $\sin\theta_{13} \simeq q(\alpha+2\beta p)/2\sqrt{2} \simeq q\alpha/2\sqrt{2} \simeq 0.14\alpha$, assuming $\beta \ll \alpha p^{-1} \sim 100\alpha$. Since all the solutions of the solar neutrino problem require $|\alpha| < 1$ in order to have small enough $\Delta m^2_\odot$ [1], the smallness of θ_{13} follows.

The seesaw mechanism we have studied naturally leads to the normal neutrino mass hierarchy while disfavouring the inverted mass hierarchy and quasi-degenerate neutrinos. For LMA and SMA solutions of the solar neutrino problem, the masses of the heavy singlet neutrinos are of the order $10^{10}-10^{11}$ GeV. For the VO solution, the lightest of the singlet neutrinos has the mass of the same order of magnitude, whereas the masses of the other two are $\sim 10^{12}-10^{13}$ GeV.

REFERENCES

1. E.Kh. Akhmedov, G.C. Branco, M.N. Rebelo, hep-ph/9911364.
2. T. Kajita (SuperKamiokande Collaboration), Talk at *Beyond the Desert'99*, Ringberg Castle, Germany, June 6-12, 1999.
3. CHOOZ Collaboration, M. Apollonio *et al.*, hep-ex/9907037.
4. E. Kh. Akhmedov, Phys. Lett. B467 (1999) 95.
5. M. Jeżabek, Y. Sumino, Phys. Lett. B440 (1998) 327.